



Article

A Cognitive Approach to Mathematical Mapping: From Classical Education to Meaningful Environmental Learning

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ABSTRACT

This study examines how integrating a cognitivist approach with the principles of classical education can promote meaningful learning and critical thinking in addressing contemporary environmental and sustainability challenges, particularly through the synergy of *MathMap* strategies. This research draws upon the liberal arts in dialogue with David Ausubel's theory of meaningful learning. This synthesis is explored for its potential to enrich pedagogical practices, anchor new knowledge in pre-existing cognitive structures, and enhance the understanding of environmental issues. Grounded in cognitivist and classical frameworks, as well as the principles of critical environmental education, the study aligns with education for sustainable development from the United Nations, which emphasizes education for sustainability through critical and reflective engagement. The findings suggest that this approach contributes significantly to the development of reflective, ethical, and critical individuals capable of navigating complex sustainability issues.

Keywords: MathMap; classical education; meaningful learning; environmental education.

RESUMO

Este estudo examina como a integração de uma abordagem cognitivista com os princípios da educação clássica pode promover a aprendizagem significativa e o pensamento crítico no enfrentamento dos desafios ambientais e de sustentabilidade contemporâneos, particularmente por meio da sinergia das estratégias do MathMap. Esta pesquisa se baseia nas artes liberais em diálogo com a teoria da aprendizagem significativa de David Ausubel. Essa síntese é explorada por seu potencial para enriquecer as práticas pedagógicas, ancorar novos conhecimentos em estruturas cognitivas preexistentes e aprimorar a compreensão das questões ambientais. Fundamentado em estruturas cognitivistas e clássicas, bem como nos princípios da educação ambiental crítica, o estudo se alinha com a educação para o desenvolvimento sustentável das Nações Unidas, que enfatiza a educação para a sustentabilidade por meio do engajamento crítico e reflexivo. Os resultados sugerem que essa abordagem contribuiu significativamente para o desenvolvimento de indivíduos reflexivos, éticos e críticos, capazes de lidar com questões complexas de sustentabilidade.

Palavras-chave: MathMap; educação clássica; aprendizagem significativa; educação ambiental.



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Introduction

The global environmental dynamic has been widely discussed and is frequently associated with development models prevalent in contemporary societies, which exert increasing pressure on natural systems (Severo et al. 2021). There is recognition of the importance of education for sustainable development (Guevara-Herrero et al. 2024), (Shulla et al. 2020), and educational systems often focus on isolated ecological content and actions disconnected from critical reflection and active student participation (Kioupi & Voulvoulis 2019). This stance ultimately hinders the development of the systemic and ethical competencies necessary to face complex contemporary environmental challenges (Giangrande et al. 2019). In this context, Howlett et al. (2016) discuss rethinking current curricula and pedagogical practices in order to promote deeper cognitive engagement. Meaningful learning offers a framework for anchoring new information to relevant pre-existing knowledge structures through cognitive processes such as *progressive differentiation* and *integrative reconciliation* (Ausubel 1963, 2003); (Ausubel et al. 1980).

Classical education in Antiquity and the Middle Ages was, traditionally structured around the seven liberal arts, divided into the trivium and the quadrivium (Lewinski 2024). Together, these liberal arts sought to form individuals capable of rigorous intellectual inquiry, coherent interpretation of knowledge, and effective articulation of ideas. Although rooted in the classical tradition, the epistemological and pedagogical principles underlying this educational model remain highly relevant to contemporary educational frameworks, particularly in fostering structured knowledge organization, logical reasoning, and meaningful learning processes (Nussbaum 2010).

Meaningful learning, when combined with the principles of classical education, particularly the trivium, promotes conceptual clarity, logical reasoning, and strong argumentative skills. These competencies, once central to education but now largely neglected, are crucial for the development of reflective, ethical, and critical individuals capable of addressing pressing environmental issues. The synergy of classical educational principles with cognitive science offers a promising pedagogical synthesis. Visual and structured learning tools, such as concept maps and *MathMap* – a computational learning tool based on the visual representation of the problem-solving process through branching structures, in which different strategies can be explored, compared, and revisited throughout the cognitive journey, favoring metacognitive reflection and the understanding of high-level strategies – can improve knowledge organization, deepen understanding, and connect abstract concepts with practical applications (Beaudouin-Lafon & Xia 2021).

These strategies not only aid in building a coherent knowledge base but also promote higher-level thinking skills, empowering students to engage in complex discussions and problem-solving related to sustainability. By combining the depth of the classical tradition with the precision of cognitive approaches, educators can cultivate both intellectual rigor and practical competence to address the pressing environmental issues of our time. This research is based on classic and contemporary frameworks related to meaningful learning, classical education, environmental education, and knowledge organization.

A comprehensive approach to mathematical mapping

Classical Education

Classical education is widely recognized for its richness and complexity, encompassing diverse dimensions of human and social development (Fernández 2022). The seven liberal arts were divided between the *trivium* (grammar, logic, rhetoric) as the art of the mind (Figure 1), and the *quadrivium* (arithmetic, music, geometry, astronomy) as the art of things (Lewinski 2024). The *trivium* encompassed knowledge related to language and thought, while the *quadrivium* was dedicated to numerical disciplines, and the union of these seven areas constituted a formative path oriented towards *kalokagathia*, that is, the integration of supreme values such as truth, goodness, and beauty into a harmonious and ethical totality (Lewinski 2024), with the goal of forming wise, virtuous, and eloquent leaders (Perrin 2018).

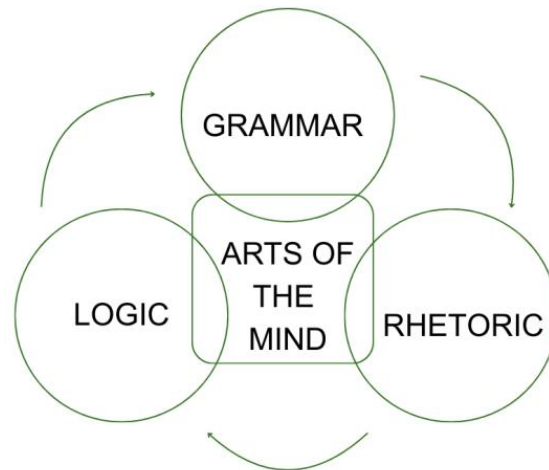


Figure 1. Diagram illustrating the trivium as the arts of the mind. The circular and interconnected arrangement highlights the dynamic and reciprocal relationship among these three domains, emphasizing their role in shaping cognitive clarity, intellectual coherence, and expressive competence. Within the framework of classical education, the trivium functions as the foundational stage of human formation. Source: prepared by the authors.

Grammar aimed to enable the clear and structured formulation of ideas; dialectics served as the basis for logical reasoning and critical analysis; and rhetoric was used to persuade and convince through articulate language. In this way, grammar organized knowledge, dialectics promoted its in-depth understanding, and rhetoric ensured its effective and persuasive transmission (Lewinski 2024). During classical antiquity, the holistic view of knowledge began to fragment, and this process considers *logos* as the reason that organizes the world and distances it from its metaphysical and universal function (Conrad 2014). Subsequently, Christianity reinterpreted *logos* as Christ, preserving part of the philosophical tradition but subjecting it to a matter of faith. With this, access to transcendence came to depend exclusively on divine grace (Conrad 2014).

The gradual fragmentation of the liberal arts reflects a shift in the conception of education, which has become primarily oriented towards economic growth (Porter 2021). Deresiewicz (2014) criticizes this model for producing technically competent professionals who are emotionally fragile and intellectually demotivated. In contrast, Nussbaum (2010) advocates an education focused on the formation of democratic citizens, supported by the humanities and moral reflection. Restoring this formative and philosophical dimension requires recovering the harmony between reason, intuition, and spirituality—essential elements for a truly holistic education (Conrad 2014). Classical education is a living tradition that needs to be revitalized after being gradually eroded by modern pedagogical reforms (Diener 2024).

The relevance of this tradition is highlighted by Leach (2024), who bases his argument on Mortimer J. Adler's view of education as the cultivation of "goods of the mind" and "goods of character," prerequisites for *Eudaimonia*. Adler criticized education limited to vocational goals, advocating instead the liberal arts and great works to develop autonomous thought and self-governance. His research revealed a widespread inability among students to think critically or govern themselves. The seven liberal arts are a bridge to moral and intellectual excellence, freeing the mind from ignorance. Wilson (2023) extends this view through Donnelly's metaphor of the trivium as "seeds" that, once sown, produce lasting intellectual and moral growth, rooting education in the love of the true, the beautiful, and the good.

Historical and literary cases illustrate the lasting impact of classical education. Lewis (2008) documents how the broad classical education of Thomas Jefferson and James Madison equipped them with strategic vision, critical thinking, and historical awareness, enabling fundamental contributions to American democracy. Similarly, Dobyns and Laskaya (2003) highlight *Le Morte Darthur* as a window into medieval pedagogy grounded in grammar and rhetoric, central disciplines for intellectual development, ethical reasoning, and persuasive discourse. In both historical and modern contexts, the trivium and quadrivium emerge as essential structures for the development of reflective, virtuous citizens capable of aspiring to environmental change. Currently, the objectives of classical education and its overall purpose vary according to each school. Even so, among the classically oriented institutions analyzed in this study, there is a recurring emphasis on character formation, clarity of expression, and the pursuit of truth, goodness, and beauty (Richard 2022).

Meaningful Learning

The cognitivist approach posits that new information is assimilated more effectively when anchored in existing cognitive structures (Ausubel et al. 1980), (Moreira 2023). This anchoring process occurs when new information is incorporated non-arbitrarily into the cognitive structure, resulting in the growth and modification of subsuming concepts (Ausubel 1963); knowledge is organized in the human brain through a hierarchy of concepts (Moreira 2023). Thus, the most general and inclusive terms of the content are presented first, and these are progressively linked to more specific terms of knowledge – a process called progressive differentiation (Figure 2). On the other hand, integrative reconciliation promotes the articulation of ideas into a coherent, effective, and meaningful system. An effective strategy for developing progressive differentiation and integrative reconciliation is the use of concept maps (Figure 2), which help visualize and organize the relationships between concepts (Moreira 2023).

The core of the learning process lies in the integration of new knowledge into the learner's pre-existing cognitive structure. Knowledge is not simply acquired through repetition or association but constructed from meaningful relationships between the new and the already known with two types of learning: rote learning and meaningful learning. The first is characterized by the literal memorization of content, generally short-term and with little transferability; the second, by the comprehensive incorporation of new concepts into previous mental schemas, which favors lasting retention and the ability to apply knowledge in new situations (Ausubel 1963). Meaningful learning is an active, not passive, process, even when it occurs through reception. Verbal instruction can be highly effective, provided it is organized according to principles of progressive differentiation and integrative reconciliation (Figure 2).

These principles guide the hierarchical structuring of content and the explication of the relationships between *new* and *old* ideas, avoiding ambiguities and contradictions (Ausubel 2003). A piece of research on how children develop their understanding of scientific concepts (Novak & Canas 2010), with analyses conducted solely through interviews, proved insufficient to accurately identify the cognitive changes involved in this process. The creation of a more effective way to graphically represent conceptual understanding took form as concept maps (Figure 2) (Novak & Canas 2010).

Concept maps consist of organizing a specific focal issue through logical structures, encompassing the concepts and relationships that underpin a system, such as conceptual hierarchy and flow models. The broadest and most comprehensive concepts are found at the top of the concept maps, while the more specific and restricted ones appear at lower levels (Novak & Canas 2010). The concept map acts as an internal *subsumer* that helps to fit new environmental propositions into a well-structured pre-existing logical network; new information only becomes fixed if it can be associated with existing concepts in a logical and coherent way, thus favoring meaningful learning (Moreira 2006). Concept maps facilitate the identification of relationships between newly acquired knowledge and previously consolidated knowledge (Van Rensburg et al. 2023).

Concept maps are tools that allow for the graphical representation of relationships between concepts structured in the form of propositions. This is a didactic resource that makes visible the meaning attributed to a given content, which favors the organization and integration of knowledge, helping the student to visualize the connections between concepts (Farias 2022). *MathMap* allows this logical visualization to be articulated with more complex relationships, such as environmental reality (Andrews et al. 2008). Using tools such as concept maps and *MathMap* in the teaching and learning process for understanding environmental complexity enables the student to direct and build a robust and logical cognitive structure (Almulla & Alamri 2021), (Schroeder et al. 2018).

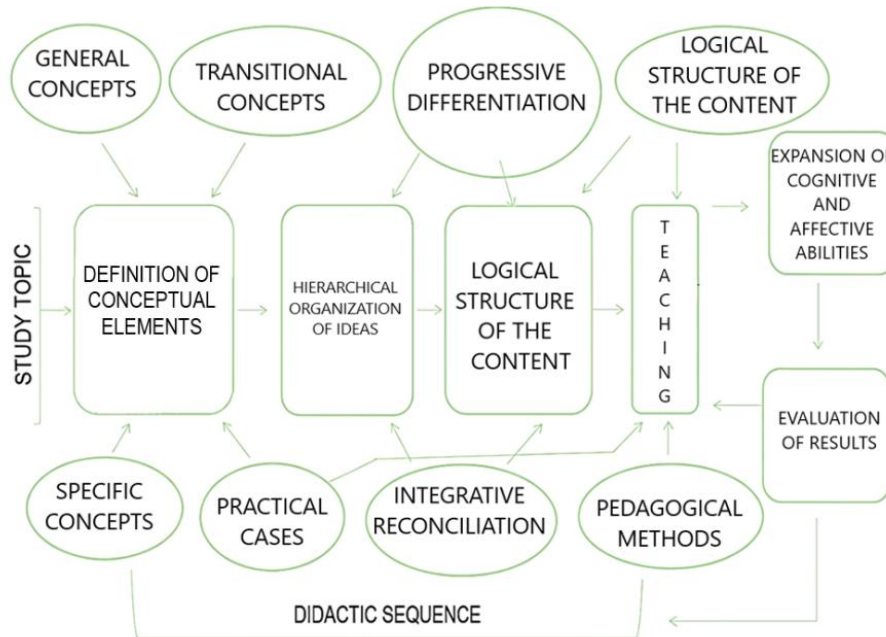


Figure 2. Conceptual organization of content for meaningful learning. This diagram illustrates a didactic sequence oriented toward meaningful learning, highlighting the systematic organization of concepts and instructional actions throughout the teaching process. The logical structuring of content is integrated with pedagogical methods, instructional strategies, and assessment processes, culminating in the development of learners' cognitive and affective abilities. Source: prepared by the authors.

Authors such as Moreira (2023) and Teixeira and Arriel (2024) provide practical guidelines for classroom implementation, highlighting their ability to enhance conceptual clarity, logical reasoning, and critical thinking. By juxtaposing cognitivism with other learning paradigms — behaviorist, humanist, and constructivist — Moreira emphasizes its unique potential to promote deep and transferable understanding (Moreira 2023); (Teixeira & Arriel 2024). Cognitivism values the organization of knowledge into mental structures (schemas), promoting critical understanding and transfer to new situations. Tools such as concept maps, advance organizers, solved examples, and metacognitive strategies are essential for integrating new concepts into existing cognitive structures, facilitating lasting and meaningful learning (Togatorop et al. 2025). Complementing this, Gowin's "Epistemological V" diagram visually connects the theoretical and methodological domains, linking abstract concepts to empirical processes (Moreira 2006). When combined with the visual-spatial organization of *MathMap*, these tools create a robust cognitive framework, allowing students to connect, anchor, and apply knowledge effectively and structurally, starting from simple concepts like digits to abstract ones like transcendental numbers and n-dimensional spaces (Figure 3).

Freudenthal (1991) presented a conception for teaching mathematics, based on the idea that students should construct knowledge progressively. From this perspective, it becomes essential to consider the different stages of students' understanding, starting with concrete experiences and gradually advancing to more abstract representations (Novo et al. 2020). This conception directly dialogues with the principles of Ausubel's theory of meaningful learning, as discussed by Moreira (2023). According to this theory, new information can only be understood and retained permanently when it finds, in the subject's cognitive repertoire, well-organized and inclusive prior concepts that allow the hierarchical integration of new knowledge with existing knowledge. *MathMap* is directly associated with the development of human cognition in the way we understand and the logic that structures mathematical thinking (Houdé & Tzourio-Mazoyer 2003).

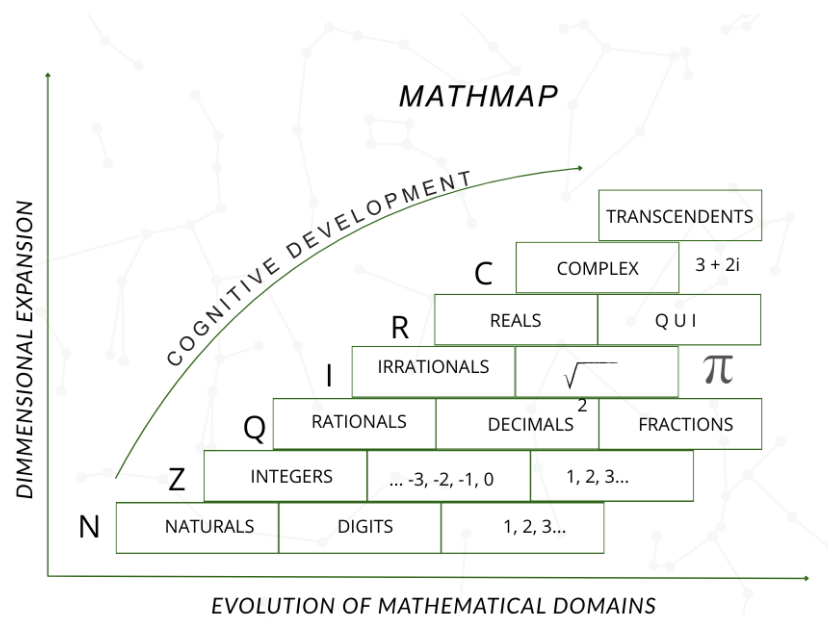


Figure 3. Hierarchical organization of mathematical domains and cognitive expansion. Beginning with basic numerical representations, such as digits and natural numbers, the diagram advances through integers, rational and irrational numbers, real and complex numbers, and culminates in transcendental constructs. This progression is aligned with a dimensional expansion that reflects the gradual development of cognitive complexity, in which each new domain is conceptually anchored in previously established structures. Source: prepared by the authors.

Studies reveal that mathematical learning permeates various cognitive systems, from simple numerical skills to complex logical reasoning as well as problem-solving techniques (Pappas et al. 2018). In their research, Träff et al. (2020) examined the cognitive models of the hierarchical development of mathematical understanding, where six-year-old children were assessed on numerical and cognitive skills before starting school. The study revealed the existence of a hierarchical structure in the development of mathematical learning. Over time, the consolidation of competencies in initial and later stages becomes progressively essential for mastery and success in later content, which presents greater complexity and abstraction (Träff et al. 2020).

A *MathMap* should be understood as an essential tool for organizing and interpreting information about the world (Gilmore 2023). The systematization of data and the creation of patterns contribute to expanding human understanding, especially with regard to the environment. From the perspective of Macias-Becerra et al. (2022), understanding the mathematical map as a human activity implies assuming a conception that positions it as a tool for interpreting phenomena that affect the planet. Furthermore, it is pertinent to consider the distinction proposed by Freudenthal (1991) between horizontal mathematization – which corresponds to the transition from the real world to the symbolic world – and vertical mathematization – which refers to operations performed within the symbolic system itself (Macias-Becerra et al. 2022). The inclusion of environmental education (EE) in mathematics teaching constitutes an opportunity to bring mathematical content closer to the student's reality, strengthening the formation of more conscious and engaged individuals within their environment (Kazay & Bredariol 2011).

Using *mind maps* as a visual tool promotes the organization of information and the externalization of prior knowledge (Karatekin 2013). Karatekin sought to understand elementary school students' perceptions of environmental problems using mind maps as a research tool, and it became evident that the environmental problems most frequently mentioned by the participants were air pollution and waste accumulation. Karatekin also observed that mind maps predominantly reflected a local perception of environmental issues, since global-scale issues, such as global warming, biodiversity loss, and soil degradation, were less frequently cited. This finding reveals a limitation in the students' awareness of more complex and comprehensive environmental problems.

Visual strategies enhance not only the retention and transfer of information, but also the ability to apply learned concepts to real-world environmental and sustainability challenges Liell and Bayer (2019) agree with the idea that by introducing environmental issues into mathematics classes, teachers not only contribute to students' understanding of different phenomena, but also favor the practical application of

mathematics in everyday environmental contexts. This approach enables students to recognize their social role and their ability to promote transformations in the reality in which they live (Su et al. 2023). Several studies corroborate the idea that including environmental issues in mathematics classes broadens students' understanding of diverse phenomena and strengthens the practical application of mathematics to everyday environmental contexts, in addition to stimulating the social role and transformative capacity of students (Bulut & Borrromeo Ferri 2025), (Klymchuk et al. 2008), (Martín-Cudero et al. 2024), (Suh & Han 2019), (Vásquez et al. 2023).

Mathematical Modeling

EE should not be understood merely as an educational modality geared towards specific purposes or as a functional instrument intended to address environmental problems (Busi et al. 2023), (Karpan et al. 2020). Rather, it is a constitutive dimension of basic education itself, indispensable to human and social development (Sonny LR 2023). The relationship established with the environment in which we live should be understood as a common space that sustains and enables life. Sauvé (2005) reformulates EE from a mere transmission of content to a relational approach that integrates the environment, identity, and culture as essential dimensions of human and social development, advocating a critical, pluralistic, and contextualized perspective. This aligns with the goal of equipping all learners with the knowledge and skills necessary to promote sustainable development.

Educational practices regarding environmental issues, in general, have prioritized the transmission of information about nature and ecological problems, neglecting aspects such as the analysis of causes, the promotion of critical thinking, and the encouragement of transformative action (Guevara-Herrero et al. 2024b). As a result, students often understand the problems and ways to address them, but do not change their daily behaviors. One of the barriers to implementing sustainable solutions lies in our limited understanding of the totality of the system in which we are embedded (Paige et al. 2008). This phenomenon has been described by some authors as superficial engagement or "complacent environmentalism" (Pérez-Martín et al. 2019 apud Guevara Herrero et al. 2023). This limitation occurs because, although environmental knowledge is recognized as fundamental, it is not sufficient to guide concrete actions; awareness and the formation of values are essential (Otto & Pensini 2017 apud Guevara Herrero et al. 2023).

Otsuka et al. (2018) postulate the creation of an environmental learning scale based on the conceptions that Japanese students have about the act of learning in contexts related to sustainability. Otsuka's research was based on classic studies on conceptions of learning, with personal beliefs and values in the field of environmental education. The research revealed that Japanese students apprehend environmental issues in three classic dimensions — learning in the environment, learning about the environment, and learning for the environment. These dimensions configure a continuous process of internalization, in which the student moves from sensory experience to reflection, culminating in conscious and socially responsible action.

Research conducted in the Brazilian context has been devoted to investigating the interfaces between Mathematics and Environmental Education. An illustrative example is the study by Junqueira and Oliveira (2025), which analyzed the articulation between Critical Mathematical Education (CME) and Environmental Education through a systematic literature review, highlighting a recurrent trend in this line of research relating the teaching of Mathematics to environmental, social, and technological issues. According to the authors, CME and Environmental Education share principles aimed at fostering a broad, critical, and action-oriented education. Both conceive teaching as an instrument for analyzing reality and for conscious intervention in the world, with the purpose of understanding and transforming it.

Given that Environmental Education seeks to promote the development of socio-environmental awareness, the use of real-world data pertaining to environmental issues, articulated with the teaching of mathematical content, enables a critical interpretation of ecological phenomena and encourages the adoption of transformative actions. Furthermore, similarly to Critical Mathematics Education, Environmental Education is committed to the formation of critical subjects and to the development of students' creative thinking. (Junqueira & Oliveira 2025).

Similarly to the aforementioned authors, Abrandt Dahlgren and Öberg (2001) also advocate for learning grounded in real-life problems. In this regard, Environmental Education faces the challenge of developing a conceptual framework that fosters dialogue between the social sciences and the exact sciences (Jacobi 2005). Barraza et al. (2017) corroborate this perspective by observing that students' levels of

reasoning are insufficient to address challenges and comprehend environmental complexities, which underscores the urgency of re-evaluating the current educational model.

Environmental education, when structured with tools that encourage a critical stance, allows for a more comprehensive view of reality and political engagement with environmental issues (Reffhaug & Lysgaard 2024). This perspective requires valuing the active participation of individuals, as well as more flexible social practices based on cooperation among the various actors involved (Howlett et al. 2016b). From the perspective of reflexive modernization, it becomes essential to confront the fragmentation of knowledge and promote a critical, politically engaged, and at the same time, reflexive approach. Thus, the environmental dimension is configured, favoring the interconnection between multiple forms of knowledge and dialogue between different areas of knowledge (Jacobi 2005).

The starting point for understanding environmental complexity is students' reflection on the problems of their local context (Jacobi 2005). Scenarios are presented and interpreted by students, who associate them with pre-existing subsumers in a brainstorming session. From this pre-existing knowledge identified in the brainstorming, it is possible to direct emerging topics towards the acquisition of meaningful learning. Jacobi (2005) also highlights the need for in-depth reflection on actions that degrade the environment, as well as greater public sensitivity to environmental issues. The risks present in society reveal the limits and consequences of social actions; this means that society itself, in generating risks, must analyze and question its own practices (Silva et al. 2024). A paradigm shift requires a transformation in the way we perceive and value the world. This transformation should guide the education of current generations not only to deal with uncertainty and the future, but also to develop complex thinking, open to transformations, diversity and uncertainties, capable of continuously constructing and reconstructing new meanings and possibilities (Silva et al. 2024).

Two examples for the development of higher-order thinking are presented by Barraza et al. (2017): when students are exposed to methodologies that encouraged their levels of argumentation and reasoning in discussions about environmental issues. The theoretical model presented by Barraza and Ruiz-Mallen emphasizes the role of education for sustainability in promoting ethical reasoning geared towards decision-making and action. This model integrates four essential elements in the reasoning process – dialogue, divergent thinking, discussion, and debate (the 4Ds) – which are considered fundamental for the development of higher cognitive skills. Guevara Herrero et al. (2023) proposed that students analyze tables and construct graphs on the consumption of pine-cones by three species. Students had to interpret trends, justify changes in populations, and argue based on evidence. The approach taken by Guevara Herrero fosters the development of individuals capable of critical analysis and autonomously formulating interpretations of reality (Guevara Herrero et al. 2023). Stimulating these activities considers the synergy between cognitivism, classical education, and *MathMap*, since logical cognitive schemas are previously constructed by students, allowing for the gradual and hierarchical restructuring of these mental models and the incorporation of more elaborate conceptions. Vasquez et al. (2022) reveals that mathematics is still poorly aligned with environmental issues and has a low presence in sustainable competencies (Coles et al. 2024).

MathMap and the environment operate according to logical principles because there are relationships of dependence and dynamic order. Spatial reasoning broadens the critical and dynamic understanding of space (Lee 2023). Environmental logic can be related, for example, in the form of propositions (if “A then B”), quantifications (the greater A, the greater B), and transitive implications (A implies B, B implies C). A practical example: *if soil pollution increases, biodiversity loss increases and ecosystem resilience decreases*. By expanding this notion, it becomes essential to develop educational proposals that encourage environmental awareness and consider the interactions between humans and nature (Lee 2023).

There is a natural logical environmental structure that can be understood by the student through analogies between conceptual diagrams and concept maps, in which it is possible to mentally visualize the implications and feedback loops as parts of a coherent whole (Hanger-Kopp et al. 2024). To internalize environmental logic, it is necessary to map ecological cycles from logical and mathematical perspectives (Houdé & Tzourio-Mazoyer 2003b). Logically understanding the natural world is not an end in itself, but a foundation for awareness and environmental education, as logical understanding allows the student to mentally see how their actions – consumption, pollution, deforestation – impact the system, and how this can favor more responsible choices (Stave & Kopainsky 2015). The *MathMap* is not merely a cognitive tool,



but an agent of transformation: by producing mental logic, it can form conscious, critical subjects capable of proposing and discussing alternatives to environmental problems (Bangboye & Avellán 2025).

Final Considerations

The synergy of a cognitivist framework with the principles of classical education provides a robust pedagogical approach for cultivating meaningful learning, critical thinking, and ethical responsibility in environmental education. Anchoring knowledge in well-structured cognitive frameworks, enriched by the rigor of the trivium and quadrivium and supported by visual tools such as *MathMap*, strengthens conceptual understanding and connects theory to practice. This synthesis addresses critical gaps in current educational models, aligning with SDG 4.7, by preparing students to engage with the challenges of sustainability through intellectual competence and moral discernment. Future studies should evaluate the implementation of the model in varied contexts to assess its effectiveness in shaping reflective, responsible, and environmentally engaged citizens.

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